QUASI-LOWPASS, QUASI-ELLIPTIC SYMMETRIC FILTER

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ABSTRACT

Levy has shown that a function developed by Acheiser/Zolotarev (LAZ) can yield a lowpass response with one large reflection ripple near dc. Rhodes and Alseyab developed a novel method of obtaining even/odd mode impedances of a symmetrical network and applied their technique to low pass filters with one transmission zero (TZ) at infinity and all others at a specified stopband frequency.

This paper combines outstanding features of these references to present a new quasi-lowpass, quasi-elliptic symmetric filter having LAZ passband response and finite stopband TZ's. Design advantages are pointed out and filter realizations are discussed.

INTRODUCTION

In the late 1800's, Zolotarev [1] extended Chebyshev's work on equal-ripple functions. In the 1920's, Acheiser [2] further developed and reported upon Zolotarev's work. In 1970, Levy [3,4] reviewed these papers and applied them to approximation of quasi-lowpass filters having mixed lumped/distributed sections. Levy collected, expounded on, and made the LAZ approximating function more widely useful to the engineering community.

In a separate investigation, Rhodes and Alseyab [5] used a simple ingenious method to obtain reactance function even/odd mode impedances of any lossless, ladder, symmetrical two-port network. Their 'alternating pole' technique was used to synthesize lowpass filters having quasi-elliptic stopband response. With all transmission zeros (TZ) at the same frequency, save one at infinity, they obtained skirt slopes much steeper than that of Chebyshev approximation, but not as steep as that of full elliptic, for a given stopband rejection level.

This paper utilizes features from these earlier works to obtain quasi-lowpass, quasi-elliptic response in a symmetric filter that is easy to realize. The response is equal ripple in the passband from λ to 1, and has one larger ripple between zero and λ as dictated by LAZ approximation. The stopband has one TZ at infinity, with all others distributed in $\pm$ pairs at finite, or infinite, freq's.

This presentation features a short review of a computer program developed to generate the LAZ function and set up the reactance function for synthesis of even/odd mode impedances. The affect of varying electrical and physical parameters will be discussed and dimensional results of varying λ and stopband TZ positions will be emphasized.

APPROXIMATION

Levy [3] shows that the Acheiser/Zolotarev (LAZ) approximating function, $f(X)=F(U)$, in the insertion loss function, $IL=1+\epsilon^2 f^2$, is given for a 2N+1 branch lowpass filter having 2N+1 TZ's at infinity by

$$f(X)=F(U)=\cosh\left((N+\tfrac{1}{2})\ln\left(\frac{H(M+U)}{H(M-U)}\right)\right) \qquad (1)$$

H is Jacobi's eta function; $M=K(\lambda,k)$ is an incomplete elliptic integral of first kind of modulus k, and $U=K(\lambda Z,k)$. Z is related to X via $Z^2=(X^2-1)/(X^2-\lambda^2)$ [3,6], $X=F/Fc$ is the real frequency variable normalized to cutoff, Fc, at high end of the equal-ripple passband, and $\lambda=sn(M)$ is normalized low-frequency cutoff below which the large ripple near dc exceeds passband ripple. Thus, the passband exists in $\lambda\leq X\leq 1$, and in a small region around zero. From (1), the characteristic equation, determining k, is $(2N+1)M=K(\pi/2,k)$, with K now a complete elliptic integral of first kind. This equation is nonlinear in λ but can be solved iteratively for k and, thus, for $M=K/(2N+1)=M1c$ for series L-shunt C ladder branches with TZ's at infinity. The Chebyshev function is a special case of f(X) for

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which $k=0$, $K=\pi/2$, $M=(\pi/2)/(2N+1)$, $\lambda=\sin(M)$ and $H(M+U)/H(M-U)=\sin(M+U)/\sin(M-U)=X+\sqrt{X^2-1}$.

The function in (1) is now extended to include P pairs of stopband TZ's along the real frequency axis, X (Fig. 1); then

$$f(X)=F(U)=\cosh\left((N+\tfrac{1}{2})\ln\left(\frac{H(M+U)}{H(M-U)}\right)+2\sum_{i=1}^{P}\ln\left(\frac{H(Mi+U)}{H(Mi-U)}\right)\right) \quad (2)$$

2P Brune sections have been added to the 2N+1 LC ladder branches, for a total network degree Nt=2N+1+4P; total distinct branches, Nd=N+1+2P; and total number of elements, Ne=2N+1+6P. The values of $Mi=K(\lambda Zi,k)$ can differ for each Brune section dependent upon the location of the stopband TZ's, Xi. The characteristic equation for this function is $(2N+1)M1c+4*\Sigma(Mi)=K(k)$, i=1, ... P, or

$$(2N+1)K(\lambda,k)+4\sum_{i=1}^{P}\left\{K\left(\lambda\sqrt{\frac{X_i^2-1}{X_i^2-\lambda^2}}\ ,k\right)\right\}=K(k) \quad (3)$$

Eqn (3) is solved iteratively by applying an arithmetic-geometric-mean subroutine to evaluate the elliptic integrals, thus obtaining modulus k and values of Mlc and Mi for LC elements and Brune sections, resp. These values, placed in Eqn (2) at Nd distinct extrema frequencies in the passband (where f(X) is pure imaginary), permit solution for Nd coefficients of Chebyshev polynomials whose sum represents the LAZ approximating function, f(X).

SYNTHESIS

Rhodes, et al, showed that $\Pi\{$LHP zeros of $1+j\epsilon f(S/j)\}=E(S)+O(S)=$constant * $\Pi\{$LHP zeros of 1+Zoo, or 1+Zoe$\}$ for a physically and electrically symmetric network. Thus, Zoe=E(S)/O(S), or O(S)/E(S), and vice-versa for Zoo. Synthesis is then initiated on one of these reactance functions once roots have been found for $1+j\epsilon f(S/j)$, their LHP terms selected and multiplied out, and the resulting even/odd parts associated with E(S) and O(S), resp.

COMPUTER PROGRAM

Equations used in developments described in the previous two sections were programmed for desktop computer evaluation, including response analyses of the resulting theoretical designs. The quasi-lowpass filter model was specialized to include any odd or even number, N, of LC ladder branches at filter input, followed by P Brune sections (each consisting of a shunt C branch followed by a parallel LC series branch), with a single shunt C branch located at center of the symmetrical half network (Fig. 2a). Input parameters to the program are:

1. Nt= total degree of the quasi-lowpass filter = 2N+1+4P
2. P = number of distinct TZ's @ finite positive freq's $\leq$ (Nt-1)/4
3. F_λ = cutoff freq (GHz) @ low end of equal-ripple passband ($\lambda=F_\lambda/Fc$)
4. Fc= cutoff freq (GHz) @ high end of equal-ripple passband
5. VSWR= voltage standing wave ratio in the equal-ripple passband
6. Fi= Brune TZ freq's (GHz) of finite stopband rejection

Selection of Chebyshev approximation can be made early in the program, permitting use of simpler calculations for this case. A plotting subroutine allows response selection from among S21amp, S21 phase, S11 return loss, group delay, and insertion loss.

COMPUTATIONAL/PHYSICAL INFERENCES

All responses shown in Figs. 3-5 were made for an Nt = 13 branch symmetrical filter with passband ripple of 0.01 dB (VSWR=1.10075). In Fig. 3, λ=0.4, all Fi=1.25, and response plots are made for P=0, 1, 2, 3. In Fig. 4, P=3, all Fi=1.25, and plots are made for λ=0.4, 0.5, 0.55, 0.6. Fig. 5 gives a similar cross section for P=3, same set of λ's, but for Fi=1.1209, 1.2697, and 2.0992 from input toward center of the filter, resulting in near-equal stopband ripple in the 86-92 dB range. Normalized computer "Gi" values are listed in Table I, from input to center of filter, for each of the cases in Figs. 3-5.

Several important suggestions can be drawn from Table I. (1) From Fig. 3, the two finite-resonant branches nearest the center have nearly identical Gi values; a slight variation in VSWR or F(1) of the input-most resonant branch would likely result in their equality. (2) From Fig. 4, near λ=0.55, Gi data shows that either all series inductors, or else all bridging series capacitors, could be made identical through slight variations in either VSWR, equal Fi locations, or by slightly separating one Fi value from the other two. (3) Gi values from Fig. 5 indicate that equal inductors of about 0.65 can be obtained for $0.55<\lambda<0.6$, with input inductor of about 0.35, for slight variations in VSWR, λ , and/or Fi. In general, capacitor values will increase and inductor values will decrease with increased λ. The flexibility of being able to set lower cutoff, λ, and stopband poles, Fi, can result in very complex filters of many branches which are relatively easy to design and realize.

EXAMPLE REALIZATION

Following suggestion (2) of the previous section, the value of λ=0.5604282, together with F(2)=1.2087 rather than 1.25, yields the set of branch Gi values below (shC, serL, shC, ..., shC; input-center):

C	L	C	L	C	L	C
0.58490	0.58178	1.5282	0.62223	2.0107	0.58178	2.3254

Note that two of the series L's are identical and the intermediate is only 6.95% greater; i.e., in the total filter, four inductors are identical and the other two are nearly so. Further, ALL capacitors bridging the series inductors are identical and of value C=1.10005. The resulting quasi-lowpass filter has 0.01 dB ripple in the normalized frequency range 0.5604<X<1 and >67-dB rejection in the stopband range 1.1<X<∞(Fig. 6). The 13-branch filter is extremely easy to realize since all series-branch elements are identical, or nearly so, and the only elements that vary are shunt C's. These shunt C's are larger than a similar Rhodes/Alseyab design, resulting in a much shorter filter. A tubular realization of the C-array for this quasi-lowpass design is very practical and easily converted to dimensions. The relatively small equal, or near-equal inductors can be housed external to the tubular structure along the length of the filter (Fig. 7).

SUMMARY AND ACKNOWLEDGMENT

A new quasi-lowpass, quasi-elliptic symmetric filter was developed theoretically using LAZ approximation. Gi values obtained by varying number, P, and location, Fi, of finite TZ's, and varying normalized low-freq cutoff, λ, permit easily-constructed filters of high performance.

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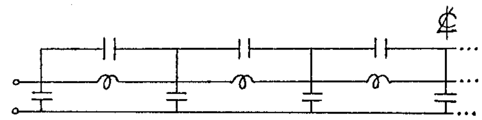
6. G. C. Temes and S. K. Mitra, Modern Filter Theory and Design, John Wiley & Sons, New York, 1973; Chs. 2, 3, esp. p. 64, pp. 114-117.

TABLE 1 Gi VALUES FOR Nt=13 SYMMETRIC HALF NETWORK



In Fig's 3&4 data below, Fi=1.25 when finite; Fi=∞ otherwise.

FIG	λ	P	N	C(1)	C(2) L(2)	C(3)	C(4) L(4)	C(5)	C(6) L(6)	C(7)
3	.40	0	6	1.146	1.230	2.711	1.066	3.851	.8547	4.450
3	.40	1	4	1.082	1.299	2.435	1.270	2.327	.9617 .6655	1.806
3	.40	2	2	1.018	1.360	1.653	.7916 .8085	1.426	.7531 .8498	1.491
4	.40	3	0	.3152	1.033 .6193	1.211	.6676 .9586	1.308	.6568 .9744	1.340
4	.50	3	0	.4598	1.051 .6089	1.377	.7886 .8116	1.621	.8328 .7685	1.722
4	.55	3	0	.5693	1.094 .5849	1.568	.9396 .6811	2.042	1.082 .5918	2.271
4	.60	3	0	.7116	1.183 .5410	1.923	1.246 .5138	2.991	1.675 .3820	3.614
F(1):F(2):F(3)=					1.12089		1.26968		2.09921	
5	.40	3	0	0	2.369 .3360	1.198	.6315 .9823	1.710	.1587 1.430	2.087
5	.50	3	0	.1601	2.262 .3518	1.323	.7454 .8321	2.194	.2137 1.062	2.887
5	.55	3	0	.2676	2.265 .3514	1.472	.8818 .7034	2.849	.2941 .7716	4.106
5	.60	3	0	.3918	2.334 .3410	1.732	1.132 .5478	4.274	.4839 .4689	7.154

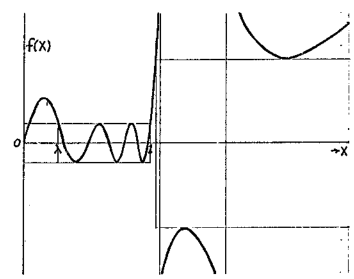


Fig. 1 - Approximating Function, f(X), for Nt=13 (P=2), (N=2)

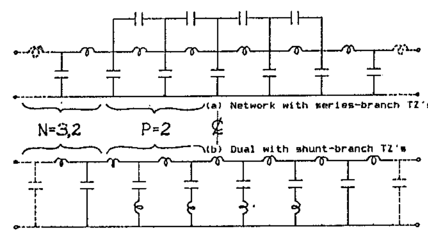


Fig. 2 - Quasi-Lowpass, Quasi-Elliptic Symmetric Filter Schematic
Nt=2N+1+4P (=15 dotted; =13 solid)

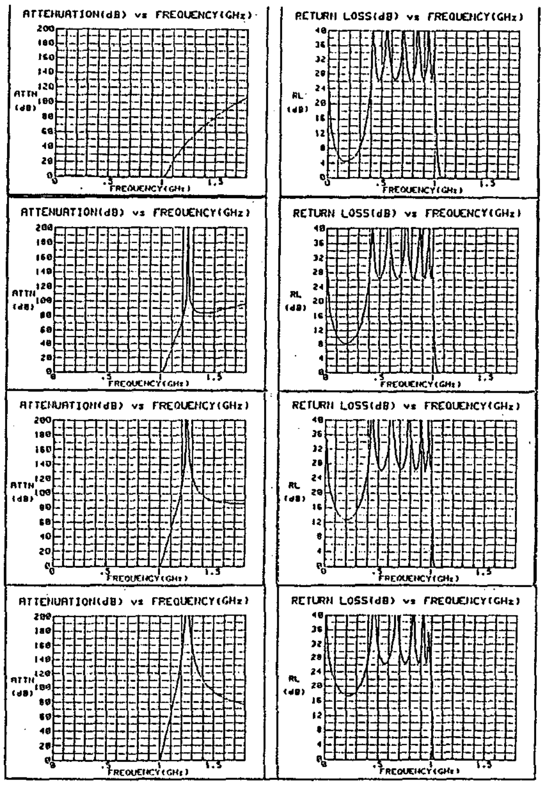


Fig. 3 Theoretical response for various P: Nt=13, λ=0.40

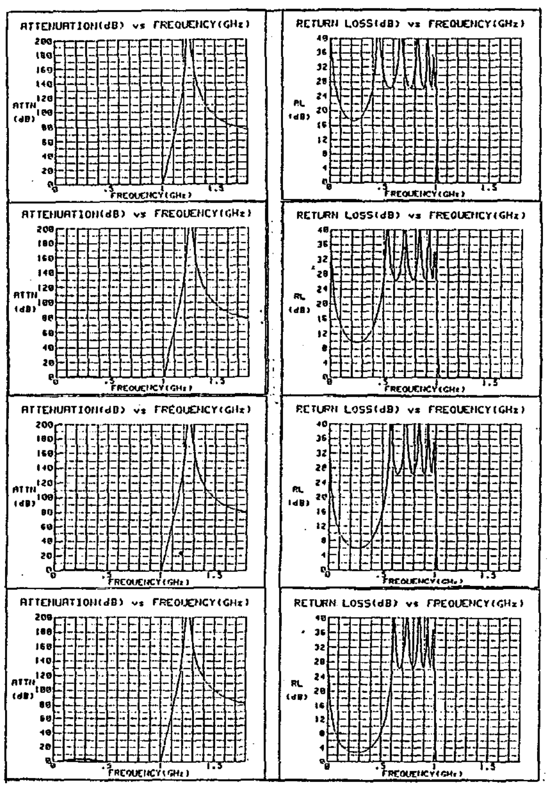


Fig. 4 Theoretical response for various λ: Nt=13, P=3
F(1)=F(2)=F(3)=1.25

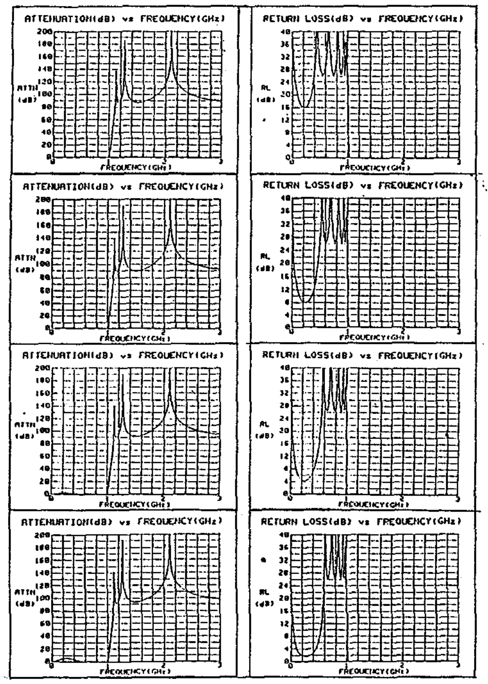


Fig. 5 Theoretical response for various λ: Nt=13, P=3
F(1)=1.1209, F(2)=1.2697, F(3)=2.0992

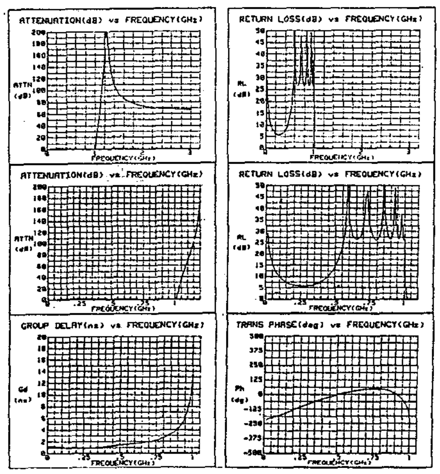


Fig. 6 Theoretical response example design : Nt=13, P=3
F(1)=F(3)=1.25, F(2)=1.2087; =0.5604282

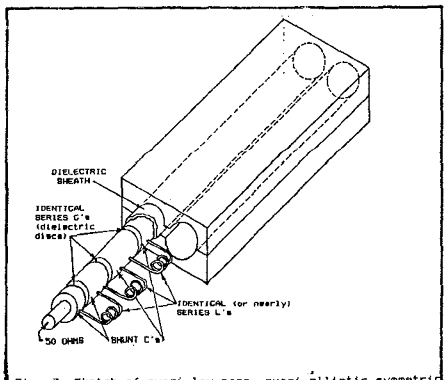


Fig. 7 Sketch of quasi low pass, quasi elliptic symmetric filter realization